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A SIMPLE MODEL**

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**1989**



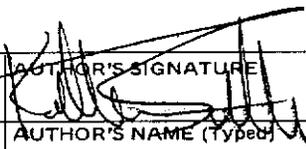
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## THE PHYSICS OF FOREST STREAM HEATING: A SIMPLE MODEL

### ABSTRACT

The basic physics of forest stream heating is investigated. Expressions for the individual energy transfer modes are developed in a simple and direct manner so that the parametric influence of various environmental conditions can be established. The environmental conditions include the daily average solar insolation, local air temperature, shading by riparian vegetation, air velocity and relative humidity, and groundwater intrusion. A mathematical model is developed and applied over a broad range of conditions. The predicted stream temperature is broken into two components, the daily mean stream temperature and the stream temperature fluctuations about the mean. The actual stream temperature is the sum of these two. Three major conclusions are drawn from the model results. First, the daily mean stream temperature is always very near the daily mean air temperature when the stream is in equilibrium with the environment. Other environmental parameters including solar insolation are shown to have relatively little **influence** on the daily mean stream temperature after an initial transient heating period. In contrast, the fluctuations in stream temperature about the mean are strongly influenced by solar insolation, riparian vegetation, and diurnal fluctuations in air temperature. Second, stream depth is the primary geometric parameter characterizing stream size for energy transfer purposes. Stream depth affects both the response time and magnitude of the fluctuations in stream temperature. Third, groundwater influx is an important **factor** in the temperature of small streams.

The full energy transfer model is then linearized so that an analytical solution is possible, and both the mean stream temperature and the fluctuating component of stream temperature are expressed as algebraic functions of only ten important environmental parameters. Comparison between the linearized model and the full model is shown to be very good. The basic results of the full model are **confirmed** by the linearized model.

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## INTRODUCTION

The temperature of natural streams is recognized to be important to aquatic life and fish productivity. This has led to studies of the response of stream temperature to environmental conditions for a variety of situations. Data reported on stream temperature for some specific situations identified key environmental characteristics important to stream temperature response for particular situations. Several studies combined field data with mathematical models. Solar radiation input has been identified as the primary environmental variable responsible for raising the temperature of small forest streams from initial groundwater temperature [Brown, 1969; Brown and Krygier, 1970; Vugts, 1974; Crittenden, 1978]. Evaporative energy loss, longwave radiation loss, and convective loss have been identified as the important energy transfer modes in cooling large streams downstream from the thermal outfall of power plants [Messinger, 1963; Ryan, 1974; DeWalle, 1976], and for being of primary importance in cooling ponds. Other studies [Raphael, 1962; Delay and Seaders, 1966; Morse, 1970; Beschta, 1984, Theurer, 1984] have incorporated all the energy transfer modes into stream network models to predict temperature of streams throughout a watershed.

The picture which emerges from these studies is that there are at least six main modes of energy transfer important in stream temperature: shortwave solar radiation, longwave radiation exchange between the stream and both the adjacent vegetation and the sky, evaporative exchange between the stream and the air, convective exchange between the stream and the air, conduction transfer between stream and the streambed, and groundwater exchange with the stream. The importance of each mode varies according to the situation. In every situation there are always several energy transfer modes involved and this makes it difficult to establish precise predictive equations for each mode for streams in natural settings.

The temperature of natural streams follows a fairly simple pattern. There is an initial transient period when the stream temperature is raised or lowered from its initial temperature to a temperature which is nearly in balance with environmental conditions. This latter situation is referred to as the equilibrium condition, though this does not imply that the stream temperature is necessarily constant. After the equilibrium condition is reached, the stream temperature is independent of the initial conditions. Overlaid on this response of the average stream temperature is a cyclic diurnal pattern due to temporal variation in solar input and air temperature. Even after the initial transient is over there are cyclic

variations in stream temperature over the course of the day. Brown [1969], Brown and Krygier [1970], and Vugts [1974] measured the temperature response of small streams during the initial transient period. Edinger, et. al. [1968] and Smith and Lavis [1975] measured the cyclic variation of small streams after the initial transient was over. Not surprisingly, the relative importance of the environmental conditions and energy transfer modes would be different for these two cases.

Large integrated computer models [Beschta, 1984; Theurer, 1984] could accommodate stream size effects, transient or cyclic/steady conditions, or environmental parameters. However, they do not allow the relative importance of the energy transfer modes to be readily identified nor the sensitivity of the results to model assumptions and formulation to be investigated.

The purpose of this paper is to develop a stream **temperature** model to investigate the basic physics of stream temperature. Each term in the energy budget will be developed in a simple and direct fashion with the intent of capturing the correct parametric influences but not the detailed site-specific data. The energy transfer terms will be incorporated into the differential equation which describes the energy budget for a stream element. This equation will be solved numerically in order to demonstrate several important features of stream temperature response. Effects of stream size and environmental conditions on transient temperature profile and diurnal variation will be demonstrated.

**Development of the numerical model** is only a **preliminary** step to the more **important and** useful linearized analytical model. To develop this, each equation describing the energy transfer modes will be linearized. The linearized differential energy budget equation will then be solved analytically to produce algebraic expressions for the transient temperature response, the daily average stream temperature, and the diurnal variation in stream temperature. The results for the linearized model will be compared to the results of the full numerical **model**.

Three significant advantages of the linearized approach will be demonstrated. The algebraic solutions to the linearized model allow ready assessment of the impact of environmental conditions and stream size on **stream** transient temperature response, daily average stream temperature, and diurnal variation in stream temperature. It also preserves the individual energy transfer exchange terms so that they may be readily modified as new and better descriptions emerge. Finally, and most importantly, for the present work, it allows

correlation techniques to be developed for portraying field data. These will be used in a companion paper “KATE SULLIVAN” to portray a vast amount of stream **temperature data** taken in the Pacific Northwest. The organized portrayal of this diverse data will help establish the validity of the modeling approach and the general validity of the expressions for **the** individual energy transfer terms.

This paper is broken into several parts. The first part describes the equations used for each of the energy transfer modes. The second part develops the energy budget equation for a stream element based on these modes and shows results from the numerical solution of the differential equation. The third part develops the linearized energy budget equation. Comparison between numerical and analytical solutions is *presented*. *The* fourth part discusses the basic physics of forest stream temperature, presents the conclusions drawn from the modeling work, and identifies methods for most clearly portraying field data.

## THE ENERGY TRANSFER MODES

### Vertical and Lateral Stream Temperature Uniformity

An important assumption that will be used in the development of the stream temperature model is that **stream** temperature is uniform in the vertical and lateral directions. This assumption of **good** mixing means that the stream top surface temperature is the same as the bulk stream **temperature**.

The level of mixing and turbulence for flowing streams depends on the relative importance of inertial forces and viscous forces. The Reynolds number is used to express this for most flow situations. Reynolds number is the ratio of inertia forces to the viscous forces for the flow. When the Reynolds number is above approximately 600 [Crittenden, 1978] the stream is in turbulent flow. Under turbulent flow conditions mixing is high and temperature gradients in the vertical and lateral directions are small. The Reynolds number is given by:

$$Re = \frac{V_s D}{\nu} \quad (1)$$

Here the stream depth,  $D$ , is taken as the characteristic dimension of the stream. The value of the kinematic viscosity,  $\nu$ , for water at 20°C is  $1 \times 10^{-6} \text{ m}^2/\text{s}$ . A relatively shallow stream 10 cm in depth with a relatively low velocity,  $V_s$ , of 10 cm/s would then have a Reynolds

number of 10,000. In general, flowing streams have Reynolds numbers very far above the threshold for turbulent mixing.

Based on the value of Reynolds number, the assumption of uniform lateral and vertical temperature is a good one for natural streams. This makes the formulation of the energy budget substantially easier and leads to two important simplifications in the formulation of the heat transfer equations for solar radiation and **streambed** conduction.

### Solar Radiation

Shortwave solar radiation heat input to a **stream** has been investigated and modeled in previous work [Anderson, 1954; Koberg, 1964, Beschta, 1984; **Theurer, 1984**]. Solar radiation arrives at the stream surface either directly or by diffuse pathways due to scatter in the atmosphere or reflection from topographic features. The only precise method of obtaining solar radiation input to a stream is to measure it for a specific site at a specific time.

Methods for estimating solar radiation input to a stream vary considerable in complexity. The important features that must be taken into account in order to estimate solar input are that: 1) solar insolation varies with geographic location, 2) it varies over the course of the day, 3) it can be reduced approximately uniformly over the course of the day by clouds, and 4) it can be blocked for portions of the day by adjacent riparian vegetation and topographic features. The approach used here accounts for each of these factors in simple (and therefore approximate) ways. The advantage sought in this formulation is ready assessment of the magnitude of each factor for a particular site and ready assessment of the predicted stream **temperature** sensitivity to each factor.

The solar insolation that would reach the stream top surface on a clear day when there is no blocking by riparian vegetation or topographic **features** is the instantaneous solar insolation. This will be calculated from the product of the peak solar insolation for the day multiplied by a time-of-the-day-factor, TODF. The peak solar flux is approximately 2.7 times the more readily available [Kreidcr and **Kreith, 1977**] average daily insolation,  $I_d$ . The value of 2.7 is based on typical solar insolation profiles [**Kreith and Black, 1980**]. For the Pacific Northwest of the U. S., the summertime average daily solar insolation is approximately 280  $W/m^2$ . This value will **be** used in some of the example cases presented below.

The clear-sky, unobstructed solar insolation calculated as described above must be multiplied by a blocking factor, BF, and a factor which shows the impact of the fraction of cloud cover, CF, to obtain the actual solar heat flux reaching the **stream**. It must also be multiplied by an effective absorptivity of the **stream** for shortwave radiation,  $\alpha_{sw}$ . When all these factors are included the expression for solar heat flux to the stream becomes:

$$q_{solar} = 2.7 I_d \alpha_{sw} (1 - 0.7 CF) (BF) (TODF) \quad W/m^2 \quad (2)$$

The cloudiness factor, CF, in Equation (2) is the fraction of sky covered by clouds. It has a value between zero and one. On a very dark, cloudy day the value of CF is zero. Solar radiation does not go to zero on these days because of the diffuse component of solar radiation. The factor 0.7 multiplying CF is an average value based measurements of the effect of cloudiness on solar insolation [Raphael, 1962]. This value is, in fact, a function of the type and altitude of the cloud cover, but these effects will not be specifically taken into account here.

Specifying CF requires judgement, but this is fortunately not very difficult for the current purposes. One of the most important periods for studying **stream** temperature is during the summer months because of potential **stream** heating and its effect on aquatic life. During this time the cloudiness factor is usually quite low at locations away from coastal areas. Second, like many other parameters, it will be shown that stream temperature predications are not very sensitive to the exact specification of CF.

The time-of-day factor, TODF, could be handled in many ways including numerical specifications of a solar insolation hourly variation. For the current purpose a simple cosine profile has been used [Edinger, et. al., 1968; Beschta, 1984] so that:

$$TODF = \cos\left(\frac{\pi}{43200} t + \pi\right) \quad (3)$$

This particular form would indicate a negative solar flux if it were not multiplied by a blocking factor, BF, which had a value of zero during the night time hours. The TODF assumes that the maximum solar insolation occurs at 12 noon. As well, the sun shines for 12 hours during the day from 6:00 to 18:00. This approximates the conditions for the summertime which is most critical for stream heating. The expression would have to be modified for other times of the year by changing the phase angle in Equation (3). Using a

value of  $5\pi/6$  for the phase angle in Equation (3) would yield a solar insolation maximum at 14:00 (2:00 p.m.), for example.

The blocking factor has two important purposes. It reduces the solar insolation to zero at night and it accounts for the blocking of solar radiation for periods during the day due to riparian vegetation and topographic features. The specific equation given below for BF makes use of a view factor of the water for the sky,  $F_{\text{wsky}}$ . In dealing with radiation heat transfer, use must be made of view factors. These specify the fraction of the total hemispherical view from the **stream** surface that is occupied by various features. The total view from the stream surface is occupied either by the sky or by the vegetation and topographic features.  $F_{\text{wsky}}$  specifies the fraction of the view occupied by the sky. For a stream on a flat plane with no riparian vegetation,  $F_{\text{wsky}} = 1$ . For a stream completely **confined** by **stream** bank and riparian vegetation, the value of  $F_{\text{wsky}} = 0$ . The view factor of the stream for the riparian vegetation and stream bank is  $1 - F_{\text{wsky}}$ .

The blocking factor, BF, uses the value of  $F_{\text{wsky}}$  to estimate the time that the sun is blocked by the riparian vegetation. Essentially this approach assumes that the stream sees the sun in the same way that it sees the sky, i. e. the sun is blocked for the same portion of the day as the view of the sky is obscured by riparian vegetation. In equation form this becomes:

$$\text{BF} = \begin{cases} 1 & \text{if } |\text{time of day} - 12| < 6 F_{\text{wsky}} \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

Note that the stream sees the sky continuously in a portion of its total view indicated by  $F_{\text{wsky}}$  while the stream sees the sun with a view factor of 1 through part of the day and with a view factor of zero for the rest of the day and night. Also note that the blocking factor is symmetric about the maximum solar insolation point of 12 noon in this formulation.

The expression for BF is only accurate for three situation, when  $F_{\text{wsky}}$  is either 0 or 1, or when the **stream** is running north-south with uniform vegetation on both sided. For all other situations there is some inaccuracy in this formulation. For the purpose of the current development this is satisfactory. One important practical situation that can be accurately assessed using this approach is the effect of complete removal of riparian vegetation from relatively small, heavily forested streams. It can, as well, show the general sensitivity of

stream temperature to various degrees of solar blocking, which is the main thrust of the present work.

The value of water absorptivity for shortwave solar radiation,  $\alpha_{sw}$ , would initially appear more difficult to specify. Several studies have shown that the absorptivity and emissivity of water for longwave radiation is very high [Koberg, 1964; Edinger, et. al., 1968; Morse, 1970; DeWalle, 1976], near 0.95. However, shortwave solar radiation is not as well absorbed by water. Measurements **confirm** the everyday experience of being able to see the bottom of the deepest swimming pool, indicating that  $\alpha_{sw}$  is considerably less than 0.95. Calculations [Crittenden, 1978] portray the absorptivity of water for shortwave radiation quite clearly. There is an initial rapid absorption of about 20% of the incident solar radiation **within** about 10 cm depth. Thereafter, absorption increases only slowly to a value near 50% at 2 m. For shallow, clear streams this would indicate relatively low absorptivity for solar radiation. However, the particular physical situation and **geometry** of streams make specification of an effective absorptivity considerably easier.

Solar radiation could penetrate a clear stream with little absorption, but would then strike the streambed surface where absorption is very high [Crittenden, 1978]. Very little of the solar radiation would be reflected by typical streambeds, particularly if they are rough or gravelly. Because the stream and streambed are both in contact with the surface of absorption, the solar energy will be split between the two. The characteristic which determines the relative split between the two is their effective thermal diffusivity [Crittenden, 1978]. When the stream Reynolds number is high, its effective turbulent thermal **diffusivity** is several orders of magnitude higher than that for any natural **streambed** material of rocks and soil [Crittenden, 1978; Kreith, 1973]. Under these circumstances the solar radiation absorbed by the streambed **surface** would be rapidly transferred to the stream, just as if it had been absorbed by the water in the first place.

Absorption by the **stream/streambed** is very high, so that the only solar insolation striking the stream surface which is not ultimately absorbed by the stream is that fraction which is reflected. During the periods of the day when solar heating is most intense the sun is high and the reflectivity is quite low, typically **about** 5% [Anderson, 1954, Raphael, 1962; Koberg, 1964, Beschta, 1984], so that the effective absorptivity is approximately 0.95. This situation is similar to that for **longwave** radiation for which the absorptivity and emissivity are approximately 0.95. This value will be used for the absorptivity and

**emissivity** of streams for all wavelengths, recognizing that it is only an effective value for shortwave solar radiation for the particular situation of natural streams.

Radiation exchange with the sky

The stream exchanges **longwave** radiation with the infrared-active constituents of the atmosphere, principally **H<sub>2</sub>O** and **CO<sub>2</sub>**. Radiation exchange with gasses in low concentrations involves very long pathlengths, often several kilometers [Hottel and Sarofim, 1967]. Such long pathlengths are rarely at uniform temperature. The **stream** looks upward and, because the temperature of the atmosphere decreases with elevation, the effective sky radiation to the stream is always lower than that for the local ground level air temperature.

Several expressions are available for the sky radiation [Anderson, 1954; Raphael, 1962; Koberg, 1964; Theurer, 1984]. These expressions can take into account both cloudiness and the relative humidity of the air. However, in the summer months when stream heating is of most concern neither of these effects is very important. All the expressions produce sky radiation that is well correlated by a simple empirical expression which has been used elsewhere [Raphael, 1962] to predict sky radiation absorbed by the stream:

$$q_{\text{sky}} = \alpha_{\text{tw}} F_{\text{wsky}} \sigma \left\{ \epsilon_{\text{sky}} T_{\text{a}}^4 - T_{\text{w}}^4 \right\} \quad \text{W/m}^2 \quad (5)$$

with

$$\epsilon_{\text{sky}} = 0.74 + 0.0049 e_{\text{a}} \quad (6)$$

This expression for  $\epsilon_{\text{sky}}$  is specifically for scattered or broken cloud conditions and not for low overcast. This is satisfactory for summertime conditions. The expression can be modified for both the elevation and density of the cloud cover [Raphael, 1962].

The value for effective sky emissivity,  $\epsilon_{\text{sky}}$ , when the partial pressure of water vapor in the atmosphere is 15 mbar is 0.81 according to Equation (6). The resulting radiation to the stream **from** the sky when the air temperature is **20°C** is equivalent to that of a surface at **5°C**.

The absorptivity of the stream water for longwave radiation,  $\alpha_{lw}$ , is approximately 0.95, as discussed above.

### Radiation Exchange with Riparian Vegetation

The stream exchanges longwave radiation with the riparian vegetation and the ground near the stream. Both are assumed to be at the local air temperature. Using the same approximation for  $\alpha_{lw}$  as for the sky radiation, the radiation exchange between the stream and the riparian vegetation is:

$$q_{veg} = \alpha_{lw} (1 - F_{wsky}) \sigma \{T_a^4 - T_w^4\} \quad \text{W/m}^2 \quad (7)$$

### Evaporation

The processes of evaporation and convective energy transfer are closely related. Both depend on transfer coefficients due to aerodynamic and buoyancy forces. Most studies have used the Bowen ratio [Bowen, 1926] to relate the two transfer coefficients which is based on the well known theoretical similarity of mass, momentum and energy transfer [Kreith, 1973]. The driving potential for evaporation is the difference in the partial pressure of water vapor in the air immediately adjacent to the stream surface,  $P(T_w)$ , and the water vapor partial pressure in the bulk air,  $e_a$ . The driving potential for heat transfer is the difference in temperature between the stream surface,  $T_w$ , and the bulk air outside the boundary layer adjacent to the stream surface,  $T_a$ . The partial pressure of water vapor adjacent to the surface is always taken as the saturation pressure of water vapor at the stream surface temperature. The bulk air water vapor partial pressure is determined from measurement of air temperature and relative humidity.

A standard for the proper height above the stream surface for measuring the bulk air conditions has not been adopted. It has been taken as high as 2m [Anderson, 1954]. This measurement height has been necessary for many previous studies of lake, pond, and stream evaporation due to the large boundary layers associated with very large open water surfaces. For most natural streams the actual boundary layer could be no more than a few centimeters [Kreith, 1973]. For the current purposes, as long as the bulk air measurements are taken above this level little error in stream temperature will be incurred

No prediction of stream temperature will be without errors. The hope is that the predicted temperatures are not very sensitive to model and coefficient assumptions. For evaporative studies the value of the transfer coefficient and bulk air conditions directly impact the predicted rate of evaporation. When predicting stream temperature, however, these same errors have considerable less impact. They do cause an error in the predicted temperature but, because the saturation pressure of water is so sensitive to stream temperature, only a very small error in stream temperature is incurred. The purpose of the linearized model and algebraic solution developed below is to be able to easily test the sensitivity of predicted temperature to assumptions and potential errors. The algebraic solution also allows alternative formulations of the transfer coefficients to be substituted for the ones selected here.

The evaporation rate can then be expressed in terms of the bulk air water vapor partial pressure,  $e_a$ , the saturation partial pressure of water vapor at the stream temperature,  $P(T_w)$ , and the evaporative transfer coefficient,  $k_e$ , which is a function of the wind velocity:

$$\text{Evaporative flux} = G_e = k_e \{ e_a - P(T_w) \} \quad \text{kg/m}^2/\text{s} \quad (8)$$

The saturation partial pressure of water vapor in the range from 274°K (1°C) to 303°K (30°C) can be calculated from the expression:

$$P(T_w) = 1.13 \times 10^{-7} \exp\{0.0653 T_w\} \quad \text{mbar} \quad (9)$$

The evaporative transfer coefficient has been the subject of many studies. It is often referred to as the velocity function. Empirically it has been found to have the form:

$$k_e = a + b V \quad (10)$$

In studies where forced convection dominates the transfer process the value of the first term is zero. This was the situation in the Lake Hefner Study [Marciano and Harbeck, 1954]. For many other situations such as cooling ponds and small streams the first term can be quite significant [Messinger, 1963; Shulyakovskiy, 1969; Ryan, 1974; DeWalle, 1976]. This term has been associated with free convection and is in fact a function of the difference in gas density between the stream surface and the air [Ryan, 1974; DeWalle, 1976; Beschta, 1984]. This term is often cast in the form of a temperature difference between the

stream surface and the air, or the same difference corrected for the partial pressure of water vapor. In either case it is raised to a low power, typically the one-third power, which is consistent with theoretical development of free convection equations. When the **temperature** difference is less than 5°C this term is very nearly constant [Ryan, 1974]. One formulation of the velocity function for conditions of mild forced convection and small temperature gradients will be used here [Shulyakovskiy, 1969]:

$$k_e = 1.74 \times 10^{-6} (1 + 0.72 V_a) \quad \text{kg/m}^2\text{/s/mbar} \quad (11)$$

### Convective Heat Exchange with the Air

The rates of convection and evaporation are **almost** always related through the **Bowen** ratio [Bowen, 1926] which has a very good theoretical base [Kreith, 1973]. The **Bowen** ratio or its equivalent [Anderson, 1954; Raphael, 1962; Fdinger, et. al., 1968; Morse, 1970, Vugts, 1974; Ryan, 1974; DeWalle, 1976] can be used to relate the convective heat transfer coefficient,  $h_c$ , to the evaporative transfer coefficient for evaporation,  $k_e$ :

$$\frac{h_c}{k_e} = 1.5 \times 10^6 \quad \frac{\text{J mbar}}{\text{kg } ^\circ\text{C}} \quad (12)$$

In the range of conditions of interest for forest streams, the convective transfer coefficient is not strongly affected by any parameter except wind velocity [McAdams, 1954; Kreith, 1973] which is taken into account directly in the expression for  $k_e$  in Equation (11).

The convective heat transfer expression is then:

$$q_{\text{conv}} = h_c (T_a - T_w) \quad \text{W/m}^2 \quad (13)$$

### Conduction to the Streambed

The stream is in contact with the **streambed** so there will be heat transfer between the two. Indeed, for porous or gravelly **streambeds**, the demarcation between the stream and the bed is not entirely clear. Energy transfer between the two can be by two entirely different mechanisms, groundwater mass transfer (which will be treated below) and by heat conduction. The impact of heat conduction to the "**streambed**" on the energy budget is not a

simple one. However, the net effect of this mechanism can, in the end, be fairly simply portrayed for the purpose of predicting stream temperature.

For the purpose of the present discussion, a non-porous **streambed** will be considered. The results of this discussion will then be broadened to conditions **where** the stream and the bed intermingle for several centimeters at their interface. When there is a non-porous interface separating the two, the **streambed** acts **like** semi-infinite solid. This configuration has been extensively treated and described in most heat transfer texts [Arpaci, 1966; Kreith, 1973]. The important feature of interaction of the streambed with the stream or with the adjacent air is that its surface is subjected to cyclic temperature variation, **both** diurnal variations and annual variations. The expression for the response of the **streambed** (or soil) as a function of time,  $t$ , and depth below the surface,  $x$ , to cyclic variations in surface temperature of the form  $[T(0,0) - T(\infty,0)] \cos\omega t$  is given in the general form:

$$\frac{T(x,t) - T(\infty,0)}{T(0,0) - T(\infty,0)} = \exp\left\{-\left(\frac{\omega}{2\alpha}\right)^{1/2} x\right\} \cos\left\{\omega t - \left(\frac{\omega}{2\alpha}\right)^{1/2} x\right\} \quad (14)$$

The thermal diffusivity,  $\alpha$ , for most non-conductors such as rocks and soil lies in the range  $1 \times 10^{-7} \text{ m}^2/\text{s}$  to  $8 \times 10^{-7} \text{ m}^2/\text{s}$  [McAdams, 1954; Kreith, 1973]. For annual variation in surface temperature, the exponential term in Equation (14) indicates that the cyclic variation in temperature at a depth of 5 m rarely exceeds 10% of the variation at the surface, except for a solid rock streambed. It does not exceed 2% for most soils and gravels. Indeed, the ground temperature at a depth of 5 m below the surface is nearly steady and constant at a value near the annual mean air temperature [Smith and Lavis, 1975], approximately 8°C (281°K) [Smith and Lavis, 1975; NOAA, 1985] for the Pacific Northwest.

The diurnal variations in surface temperature are faster than the annual variation so their effect is felt only at shallower depths. At a depth of only 30 cm, Equation (14) indicates that temperature variations are damped down to only 5% of the diurnal surface temperature variation. This is very similar to other cyclic heat transfer situations [Gorog, 1982]. Heat transferred to the bed during periods of the day when stream temperature is above the daily average air temperature is transferred back to the stream during the periods when diurnal variation brings the stream below the daily average air temperature. This has the effect of dampening the diurnal stream temperature variations without affecting the daily average **stream** temperature. Effectively this causes the stream to act as if it is deeper and contains more thermal inertia.

Though the expression presented in Equation (14) is specifically for a non-porous **streambed**, because of the cyclic variation in temperature due to diurnal changes results in a coupling of stream and bed, the same general conclusion can be drawn for loose or porous streambeds. The first 30 cm or so below the initial interface. of stream and bed will act **like** a thermal flywheel for diurnal variation. **The** temperature variations of the main flowing **body** of the stream will be damped by conduction and fluid exchange with the water below the **streambed** surface. One method of handling this effect while avoiding the cumbersome formulation of Equation (14) is to assign a **constant** temperature to the streambed and allow heat transfer to occur between **it** and the cyclicly varying stream over the course of the day. This transfer is driven by the temperature difference between the stream and the bed through an effective transfer coefficient,  $h_{sb}$ . **In** this way the stream transfers heat to the bed when its temperature is above that of the bed and vice versa.

Selecting an appropriate value for both the effective **streambed temperature** and the effective transfer coefficient must capture both the short term diurnal interaction as well as the longer annual variations. Aside from the cyclic effects there is a net **transfer** between the **stream** and the deeper soil. Over the course of a year this transfer is **also** cyclic, and therefore net zero. However, over the usual periods of measurement from a few hours to a few months there can be a net interaction. The **streambed** or soil at a depth of 5 m is at a nearly constant temperature, near 8°C for the Pacific Northwest. Because this is substantially below the average daily stream temperature for the summertime there is a net heat loss to the streambed during this period. As will be shown below, this is quite a small term in the energy budget.

The approach to selecting  $h_{sb}$  and the effective temperature is to first recognize that the daily average surface temperature of the ground is nearly the same as for the air. The peak daily average value occurs in the summertime. For the Pacific Northwest in the summertime the average daily air temperature is approximately 18°C. The Equation (14) must be **differentiated** and multiplied by thermal conductivity to obtain an equation for heat flux:

$$q = -k \frac{dT}{dx} \tag{15}$$

which at **peak** summer conditions ( $t=0$ ) and at the surface ( $x=0$ ) becomes:

$$q = k \left( \frac{\omega}{2\alpha} \right)^{1/2} \{ T(0,0) - T(\infty,0) \} \quad (16)$$

The net heat flux into the ground can be estimated by using the annual average and peak values of air temperature and a typical value of soil thermal diffusivity,  $2 \times 10^{-7} \text{ m}^2/\text{s}$ , and thermal conductivity,  $0.75 \text{ W/m } ^\circ\text{C}$  [McAdams, 1954; Kreith, 1973]:

$$q \cong 3.4 \text{ W/m}^2 \quad (17)$$

This is quite a **small** value compared to average daily clear sky solar radiation of  $280 \text{ W/m}^2$ , for example. This effect is **small** enough to be ignored in the energy budget equation.

The **term** in **front** of the temperature difference in Equation (16) is the effective heat transfer coefficient,  $h_{sb}$ . For the cyclic diurnal variations, its **value** using the property **values** in the example above is  $6.7 \text{ W/m}^2 \text{ } ^\circ\text{C}$ . The stream transfers heat to the bed through this effective value. The driving potential is the temperature difference between **the** stream temperature and the temperature about 30 cm below the surface. Using Equation (16) and the typical values for average **peak** summer temperature,  $18^\circ\text{C}$  and the average annual temperature,  $8^\circ\text{C}$ , the temperature at 30 cm is about  $16^\circ\text{C}$ , or  $2^\circ\text{C}$  below the surface temperature. The streambed heat transfer can then be approximated as the transfer through the transfer coefficient,  $h_{sb}$ , driven by a difference between the stream **temperature** and a temperature approximately  $2^\circ\text{C}$  below the average daily air **temperature**. In equation form: ,

$$q_{sb} = h_{sb} \{ \bar{T}_a - 2 - T_w \} \quad (18)$$

### The Net Heat Flux:

The net heat flux is just the sum of the individual heat flux terms:

$$q_{net} = q_{solar} + q_{sky} + q_{veg} + q_{conv} + q_{sb} \quad (19)$$

### Groundwater Influx:

Groundwater enters the stream along its length. The quantity of groundwater will depend on the time of year, geological factors, and the watershed area. The measure of

groundwater inflow used here will be a flux term equal to the rate of inflow divided by the stream top surface area. It is calculated from the actual groundwater inflow in a given length of stream divided by the total top surface area of the stream for that length. The symbol for the groundwater flux is  $G_{gw}$ . This basis for specifying groundwater input is strictly for convenience in formulating the energy budget equation.

In the example cases below the groundwater influx specified is approximately equivalent to a 1% increase in the stream mass flow per kilometer of stream length for a 0.3 m deep stream flowing with a velocity of 0.17 m/s. The range covered in subsequent examples is from 0.1% to 10% increase per kilometer of length.

### THE ENERGY BUDGET FOR A STREAM ELEMENT

The energy balance for the stream can be written in several ways with control elements that are stationary or that follow a specific mass of fluid. The control element chosen here follows an initial fluid element and lets it expand in size as groundwater enters it. The energy budget equation is:

$$\frac{d(M h_w)}{dt} = M \frac{d(h_w)}{dt} + h_w \frac{d(M)}{dt} = q_{net} A + G_{gw} A h_{gw} + G_e A h_e \quad (20)$$

Recognizing that the rate of change of mass of the element with respect to time is just the the groundwater **inflow** and the evaporation, this equation can be rewritten as:

$$\frac{d(h_w)}{dt} = \frac{q_{net} A + G_{gw} A (h_{gw} - h_w) + G_e A (h_e - h_w)}{M} \quad (21)$$

The mass of the element,  $M$ , is not constant in this expression if there is groundwater **intrusion** and evaporation. However, because the element mass and surface area appear as a ratio in this expression, a **simplification** is possible. The mass and surface area of the **stream** element are related by the expression:

$$M = \rho A D \quad (22)$$

Using this and substituting common expressions for the enthalpy **terms** yields:

$$\frac{d(T_w)}{dt} = \frac{q_{net} + G_{gw} C_w (T_{gw} - T_w) + G_e L_v}{\rho C_w D} \quad (23)$$

Two observations can be made from this equation. First, because the usual summertime stream temperatures are well above the 8°C (281°K) typical of groundwater, groundwater influx can have an important depressing effect on stream temperature. Second, the most important stream geometric characteristic which dictates its effective size is the stream depth.

### FULL MODEL PREDICTED RESULTS

The energy budget expressed in Equation (23) can be easily programmed and solved for a wide variety of conditions. Predicted results for three cases are shown in Figures 1 through 3. The basis for these calculations is given in Table I below. The three cases represent three different stream sizes of depths: 0.1 m, 0.3 m, 1.0 m. The initial stream temperature was arbitrarily specified at 5°C (278°K) in order to show the time response of stream temperature to external condition. Solar insolation was taken for a clear day in mid-July, and the air temperature profile was prescribed as a simple cosine function based on actual measured profiles at a specific site near Mt. St. Helens for a mid-July day. It was selected because of the wide fluctuations in air temperature at this site. Other values are given in the table. The calculations were carried out for a ten day period. This means that the stream element was exposed to the same environmental conditions for ten 24-hour periods in order to see the cumulative effect of many hot days in a row.

TABLE I  
Environmental Parameters for the Example Case

Mean air temperature, °K	$\bar{T}_a$	289°K (16°C)
Air temperature fluctuations, °C	$T'_a$	12
Daily average solar insolation, W/m <sup>2</sup>	$I_d$	280
Cloudiness, unitless	CF	0
View factor water-to-sky, unitless	$F_{wsky}$	0.5
Air velocity, m/s	$V_a$	0.5
Water vapor in air, mbar	$e_a$	9
Stream depth, m	D	0.1-0.3-1.0
Groundwater influx, kg/m <sup>2</sup> ·s	$G_{gw}$	0.0005
Groundwater temperature, °K	$T_{gw}$	281°K (8°C)

The results shown in these three figures have a particular pattern which is true for **all** the runs carried out with this model. The stream temperature rises in a cyclic fashion and eventually establishes an equilibrium with the surrounding environment. The time to rise to this equilibrium is less than a day for small (i.e. shallow) streams, to several days for large (i.e. deep) streams. After reaching equilibrium, the diurnal fluctuations are smaller for the deep stream than for the shallow stream, and for all cases except streams shallower than 0.1 m, the diurnal stream temperature variations are less than the diurnal variations of the air temperature. As well, at the equilibrium the daily mean water temperature is very near the daily mean air temperature.

These results are not surprising. The energy loss from the stream due to evaporation and sky radiation increases rapidly with stream temperature. Evaporation loss increases exponentially with stream temperature. As the stream is heated by solar radiation and convection, a stream temperature will always be reached where the energy losses will balance the energy gain for the day. This is demonstrated for one equilibrium condition in the bar graph of Figure 4. When the stream is being heated to the equilibrium as in the early portions of Figures 1 and 2, the solar term dominates the energy budget, as found in actual measurements [Brown, 1969; Brown and Krygier, 1970].

Most of the exchange terms involve the local air temperature so this temperature will be very influential in determining the equilibrium stream temperature. The response of the **stream** temperature to external conditions such as diurnal air temperature fluctuations is damped by the thermal inertia of the stream. Deeper streams have more thermal inertia so they heat up to **equilibrium** more slowly and fluctuate less at equilibrium.

The **stream** temperature model has been applied over a wide **range** of conditions to establish the parametric influence of various environmental conditions on the predicted results for equilibrium conditions. A small portion of the results are shown in Figure 5. This plot of predicted daily mean stream temperature versus the prescribed daily mean air temperature resulted from changing the input variables through the ranges shown in Table II.

TABLE II  
Range of Variation of the Environmental Parameters for Figure 5

Mean air temperature, °C	$\bar{T}_a$	8-16-24
Air temperature fluctuations, °C	$T'_a$	8-12
Daily average solar insolation, W/m <sup>2</sup>	$I_d$	200-280
Cloudiness, unitless	CF	0.0-0.5-1.0
View factor water-to-sky, unitless	$F_{wsky}$	0.1-0.5-0.9
Air velocity, m/s	$\bar{e}_a$	0.1-0.5-2.0
Water vapor in air, mbar	D	0.1-0.3-1.0 s-9-15
Stream depth, m		
Groundwater influx, kg/m <sup>2</sup> ·s	$G_{gw}$	0.0002-0.0005-0.002
Groundwater temperature, °C	$T_{gw}$	8

The pattern that emerges here is that the daily mean water temperature is always near the mean air temperature under equilibrium conditions. At low temperatures the water is somewhat above the air, and for high temperatures the water is usually below the air temperature. Considering the broad range of input conditions, there is a relatively small response of the daily average stream temperature at equilibrium to environmental conditions other than the local mean air temperature. This merely indicates that the energy losses **from** the stream change very rapidly with stream temperature so that large changes in the environmental conditions listed in Tables I and II are compensated by relatively small changes in mean stream **temperature**. Environmental conditions affect the stream temperature fluctuations more than the daily mean stream temperature. Because of the interactions of several variables it is more difficult to establish the influence of any one of them on the **stream** temperature fluctuations from numerical results.

For any given set of environmental conditions there will always be some mean air temperature for which the mean stream temperature and mean air temperature are equal. This is demonstrated in the plot of the  $\bar{T}_w$  versus  $\bar{T}_a$  shown in Figure 5 for the example conditions. Also shown in this figure is that the slope of this curve is less than one. For a wide variety of conditions the slope of the  $\bar{T}_w$  versus  $\bar{T}_a$  line is between 0.5 and 1.0. It should also be noted that because the groundwater is at a temperature of only about 8°C (281%) it tends to flatten the slope.

These model predictions could be used to predict the stream temperature for a watershed system. However, the real utility of this type of model is in establishing the key physical parameters important to stream heating and in identifying the influence of environmental

conditions on the **stream temperature** fluctuations. This will be the purpose of the linearized model developed below.

## THE LINEARIZED STREAM HEATING MODEL

Many of the energy flux **terms** are not linear in the value of stream temperature. However, each can be linearized about a specific condition and the resulting equations are presented below.

### Sky:

The sky and vegetation radiation terms can be linearized by algebraically expanding the fourth power differences, recognizing that the absolute temperatures for the air and water change by only a **small** percentage, and that both are approximately equal to the daily average air temperature. When this is done the radiation exchange with the sky becomes:

$$q_{\text{sky}} = \alpha_{lw} F_{\text{wsky}} \sigma \bar{T}_a^3 \left[ \sqrt[2]{\epsilon_{\text{sky}} + 1} \right] \left[ \sqrt[4]{\epsilon_{\text{sky}} + 1} \right] \left[ \sqrt[4]{\epsilon_{\text{sky}}} T_a - T_w \right] \quad (24)$$

For the usual range of values for  $\epsilon_{\text{sky}}$  between 0.78 and 0.85, this equation can be simplified to the following with very little error:

$$q_{\text{sky}} = \alpha_{lw} F_{\text{wsky}} \sigma \bar{T}_a^3 3.7 \left[ \sqrt[4]{\epsilon_{\text{sky}}} T_a - T_w \right] \quad (25)$$

### Vegetation:

The same procedure can be used for the radiation exchange with the **riparian** vegetation with the result that:

$$q_{\text{veg}} = \alpha_{lw} (1 - F_{\text{wsky}}) \sigma 4 \bar{T}_a^3 [T_a - T_w] \quad (26)$$

### Convection:

$$q_{\text{conv}} = h_c (T_a - T_w) \quad (27)$$

Evaporation:

$$G_e = k_e \left[ e_a - 1.13 \times 10^{-7} e^{0.0653(\bar{T}_a)} \left\{ 1 + 0.0653 (T_w - \bar{T}_a) \right\} \right] \quad (28)$$

Streambed Conduction:

$$q_{sb} = h_{sb} (\bar{T}_a - 2 - T_w) \quad (29)$$

Solar Radiation

Both the major driving forces for stream heating, solar radiation and air temperature, cycle over the course of a day and cause the stream temperature to cycle. They can each be expressed as the sum of a steady component and a fluctuating component. The solar radiation becomes:

$$q_{solar} = \bar{q}_{solar} + q'_{solar} \cos\left(\frac{\pi}{43200} t + \pi\right) \quad (30)$$

with the steady solar component:

$$\bar{q}_{solar} = I_d \alpha_{sw} (1 - 0.7 CF) F_{wsky} \quad (31)$$

and the fluctuating solar component:

$$q'_{solar} = 1.7 \bar{q}_{solar} \quad (32)$$

The fluctuating component for the solar radiation is, in fact, the half-wave amplitude, as is the fluctuating air temperature given below and the resulting water temperature fluctuations presented in the examples.

### Air Temperature

The air temperature is represented as:

$$T_a = \bar{T}_a + T'_a \cos\left(\frac{\pi}{43200} t + \pi\right) \quad (33)$$

### The Linearized Energy Budget Equation

The basic energy equation is the same, but using the linearized terms it can be rewritten in the form:

$$\frac{d(T_w - \bar{T}_a)}{dt} + U(T_w - \bar{T}_a) = S + F \cos\left(\frac{\pi}{43200} t + \pi\right) \quad (34)$$

with:

$$u = \frac{\alpha_{lw} \sigma \bar{T}_a^3 (4 - 0.3F_{wsky}) + h_c}{\rho C_w D} + \frac{h_{sb} + G_{gw} C_w + 7.38 \times 10^{-9} L_v k_e \exp\{0.0653 \bar{T}_a\}}{\rho C_w D} \quad (35)$$

$$S = \frac{\bar{q}_{solar} + \alpha_{lw} F_{wsky} \sigma 3.7 \bar{T}_a^4 \left[ \sqrt[4]{\epsilon_{sky} - 1} \right]}{\rho C_w D} + \frac{L_v k_e [e_a - 1.13 \times 10^{-7} \exp\{0.0653 \bar{T}_a\}] - 2 h_{sb} + G_{gw} C_w (T_w - \bar{T}_a)}{\rho C_w D} \quad (36)$$

$$F = \frac{q'_{\text{solar}} + T'_a \left[ \alpha_{lw} \sigma \bar{T}_a^3 \left\{ 4 - F_{\text{wsky}} \left( 4 - 3.7 \sqrt[4]{\epsilon_{\text{sky}}} \right) \right\} + h_c \right]}{\rho C_w D} \quad (37)$$

Physically, the term  $U$  is an effective overall energy transfer coefficient,  $S$  is the steady component of the energy flux input, and  $F$  is the fluctuating component of the energy flux input. Though each of these consists of several terms, they require only the ten basic parameters listed in Table I to evaluate.

For reference, the linearized energy budget has been evaluated for the case of an 0.3 m deep stream. The results from the linearized model and the full model are shown in Figure 6. Despite the approximations used in linearizing and in characterizing solar radiation and air temperature, the comparison is very good. Analytical solutions of Equation (34) are available in many heat transfer texts [Arpaci, 1966]. The solutions indicate that after the initial transient response has died out, the equilibrium values for the mean and fluctuating components of stream temperature can be written:

$$\bar{T}_w = \bar{T}_a + \frac{S}{U} \quad (38)$$

$$|T'_w| = \frac{F}{U} \frac{1}{\left[ 1 + \left( \frac{\pi/43200}{U} \right)^2 \right]^{1/2}} \quad (39)$$

In order to clarify the magnitude of the various items, numbers from the 0.3 m deep stream example case have been used to evaluate the terms in the expressions:

$$U = \frac{5.01 + 3.55 + 6.70 + 2.09 + 6.69}{1.26 \times 10^6} = 1.91 \times 10^{-5} \quad (40)$$

$$S = \frac{133 - 41.0 - 50.5 - 13.4 - 16.7}{1.26 \times 10^6} = +0.905 \times 10^{-5} \quad (41)$$

$$F = \frac{226 + 101}{1.26 \times 10^6} = 2.60 \times 10^{-4} \quad (42)$$

so that:

$$\bar{T}_w = \bar{T}_a + \frac{(0.905 \times 10^{-5})}{1.91 \times 10^{-5}} = \bar{T}_a + 0.47^\circ\text{C} = 16.47^\circ\text{C} \quad (43)$$

$$T_w' = \frac{2.60 \times 10^{-4}}{1.91 \times 10^{-5}} \left\{ \frac{1}{\left[ 1 + \left( \frac{\pi/43200}{1.91 \times 10^{-5}} \right)^2 \right]^{1/2}} \right\} = 13.6 \times 0.254 = 3.46^\circ\text{C} \quad (44)$$

This value of  $\bar{T}_w$  is very close to the actual value calculated by the full model of 17.5°K. The value for  $T_w'$  is also close to the full model value of 4.8°C. Note in Equation (41) that the solar input is balanced by sky radiation loss and evaporation. In Equation (42) however the solar radiation fluctuation over the course of the day accounts for almost 60% of the water temperature fluctuation.

#### LINEARIZED MODEL STREAM TEMPERATURE PREDICTIONS

The linearized model results compare well with the full model results for the example case, and this is generally true. Comparisons of the predicted results from the two models are shown in Figures 7 and 8 for the mean stream temperature and the fluctuating component of the stream temperature. Figure 7 indicates very good agreement over a wide range of conditions for the mean temperature. Not surprisingly the fluctuating component is not quite as good but still very acceptable, particularly in light of the ease of evaluation of the algebraic expressions compared to the numerical solution.

The effect of shading by riparian vegetation is different than the effect of clouds. The view factor  $F_{wsky}$  appears in two terms in Equation (36): solar radiation and sky radiation. These two terms are both directly affected by  $F_{wsky}$ , but they have opposite effects. The sky radiation always reduces the impact of solar radiation on the daily mean stream temperature. The net effect of changing the view of the stream for the sky from 90% to 10% is:

$$\bar{T}_w = \bar{T}_a + 3.6^\circ\text{C} \quad @ F_{wsky} = 0.9 \quad (45)$$

$$\bar{T}_w = \bar{T}_a - 2.6^\circ\text{C} \quad @ F_{wsky} = 0.1 \quad (46)$$

The maximum difference in daily mean stream temperature between nearly complete riparian coverage and almost no coverage is approximately 6.2°C. This assumes, of course, that all other conditions remain the same.

The first fraction on the RHS of Equation (39) accounts for the maximum possible stream temperature fluctuations of the very smallest stream. The second fraction reduces this maximum as stream size (depth) increases. The value of the first fraction in the example case is 13.6°C. This indicates that the maximum fluctuation of the stream temperature for the very smallest streams would be slightly greater than the fluctuation in air temperature which is 12°C. In the expression for  $T_w$  the solar insolation is no longer reduced by the sky radiation affects so that  $F_{wsky}$ , and therefore riparian vegetation, will have a stronger impact on the fluctuating component of stream temperature.

Deeper streams will always have smaller fluctuations than smaller streams. The impact of stream depth on the size of the stream temperature fluctuations is shown in Figure 9. Not surprisingly, the deeper the stream the smaller the fluctuations.

## DISCUSSION

A stream temperature model was developed specifically to examine the basic physics of stream heating. The model was linearized in order to obtain algebraic expressions for the basic components of stream temperature: the mean stream temperature and the stream temperature fluctuations. Both of these components can be expressed in terms of algebraic equations involving ten basic environmental parameters. This allows direct and simple evaluation of both components once the environmental factors are specified. The algebraic equations also allow simple evaluation of expected results of stream environment changes, and provide correlating parameters for a very broad range of streams with disparate environmental conditions. Some of the expected **trends** have been displayed here, and the magnitude of each term has been evaluated for one case.

Through the process of linearization and evaluation it becomes obvious that the local air temperature is the single most important parameter influencing **the** daily mean stream temperature at equilibrium. It is also apparent that solar insolation is less important except for its influence on local air temperature. Direct solar radiation to the stream surface has relatively small impact on the mean stream temperature at equilibrium. This is primarily

because of the very large increase in energy loss due to evaporation and sky radiation as stream temperature increases. The real **influence** of solar radiation is on the magnitude of the stream temperature diurnal variations. For the case evaluated here the solar insolation was responsible for about **60%** of the stream temperature variations. Even with constant mean air temperature, the solar insolation will raise the maximum **daily** stream temperature.

The intention of the current stream temperature model is to understand the physics of **stream** heating rather than to predict the temperature of a specific stream system. To accomplish this the individual environmental parameters have been cast so they can be manipulated individually to evaluate their **influence**. In a real **stream** situation any physical change is **likely** to change more than one of the environmental parameters. For example removing the riparian vegetation from a stream will increase the view of the stream for the sun and the sky, and directly affect  $F_{wsky}$ . However, it is also likely that the local air temperature, air velocity, and local relative humidity **will** change. It would be mandatory to properly adjust all three of these in the model to correctly evaluate the impact of removal of riparian vegetation on stream temperature.

It is equally important to recognize that most of the exchange terms are based on local values of each environmental parameter. It is particularly important to evaluate the air temperature and the relative humidity in the immediate vicinity of the stream. The linearized model clearly indicated the very dominant role of local air temperature on stream temperature. The use of remote or approximate values for air temperature can be expected to produce remote or approximate stream temperature predictions.

Though the air temperature and its fluctuations were treated as independent variables here, it may be one of the most important results of this work that understanding the impact of forest practices on local air temperature is a crucial step in understanding equilibrium stream temperature. Solar radiation will only dominate the energy budget **while** the stream is being heated from groundwater temperature to equilibrium conditions.

In a companion paper, "KATE SULLIVAN", a vast amount of stream temperature data from a wide variety of streams is presented. The basic form of the presentation and the correlating techniques employed follow **from** the results presented here. The data **will be** used to confirm the basic results of the model work.

## CONCLUSIONS

The individual energy flux terms and the overall energy balance for a forest stream have been modeled in a direct and simple manner. This model has been exercised over a range of conditions to gather some insight into basic **stream** heating physics. This **model** was then linearized in order to derive algebraic expressions for both the daily mean stream temperature and the stream temperature fluctuations in terms of the basic environmental parameters. Linearization of the stream heating model allows an analytical solution of the governing differential equation for stream heating **with** only very minor loss in predictive accuracy. Based on the linearized model both the daily mean value and fluctuating component of stream temperature can be evaluated with algebraic expressions involving only ten important environmental parameters. Model results suggest several general conclusions regarding the relationship of stream temperature to important environmental parameters.

**Stream Depth.** Stream depth is the most important **geometric** parameter which characterizes stream size for energy transfer purposes. Stream depth affects both the magnitude of the stream temperature fluctuations and the response time of the stream to changes in environmental condition. Streams of finite depth always show temperature **fluctuations** which are smaller than the diurnal fluctuations in air temperature.

**Air Temperature.** The daily mean stream temperature under equilibrium conditions is always very near the daily mean air temperature. The linearized model result confirms that the mean **stream** temperature will always be very near the mean air temperature and that the fluctuating stream temperature component will always be strongly influenced by the fluctuating air temperature and solar insolation fluctuation. Other environmental parameters have relatively little influence on mean stream temperature because the two major energy loss terms, sky radiation and evaporation, depend so strongly on stream temperature. Because of the dominant role of air **temperature** on stream temperature it is crucial to use the local value of air temperature to evaluate stream heating; this is also true for local air velocity and relative humidity.

**Riparian Vegetation.** Removal of riparian vegetation has relatively modest impact on the mean stream temperature because the energy gain due to increased solar radiation influx is partially offset by increased energy loss by radiation to the sky. The fluctuating stream

temperature component will be more strongly influenced by the removal of riparian vegetation because of the effect of **solar** insolation fluctuation.

Groundwater. Since the usual summertime stream temperatures are well above the 8°C typical of groundwater, groundwater influx can have an important depressing effect on stream temperature. This effect will **depend** on the **rate** of groundwater influx relative to the volume of flow in the **stream**, and on the groundwater temperature compared to the mean stream temperature.

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## NOMENCLATURE

A = stream top surface area,  $m^2$

a,b = constants

BF = blocking factor of solar radiation by **riparian** vegetation, a fraction (see Equation 2)

CF = cloudiness factor, 0 if clear and 1 if completely clouded over, a fraction

$C_w$  = heat capacity of water, 4186 J/(kg °C)

D = stream average depth, m

d = differential operator

ea = partial pressure of water vapor in air, mbar

F = fluctuating energy flux factor, °C/s

$F_{wsky}$  = view factor of the stream water for the sky, a fraction

$G_e$  = evaporative mass flux,  $kg/(m^2 s)$

$G_{gw}$  = ground water influx per unit of stream surface area,  $kg/(m^2 s)$

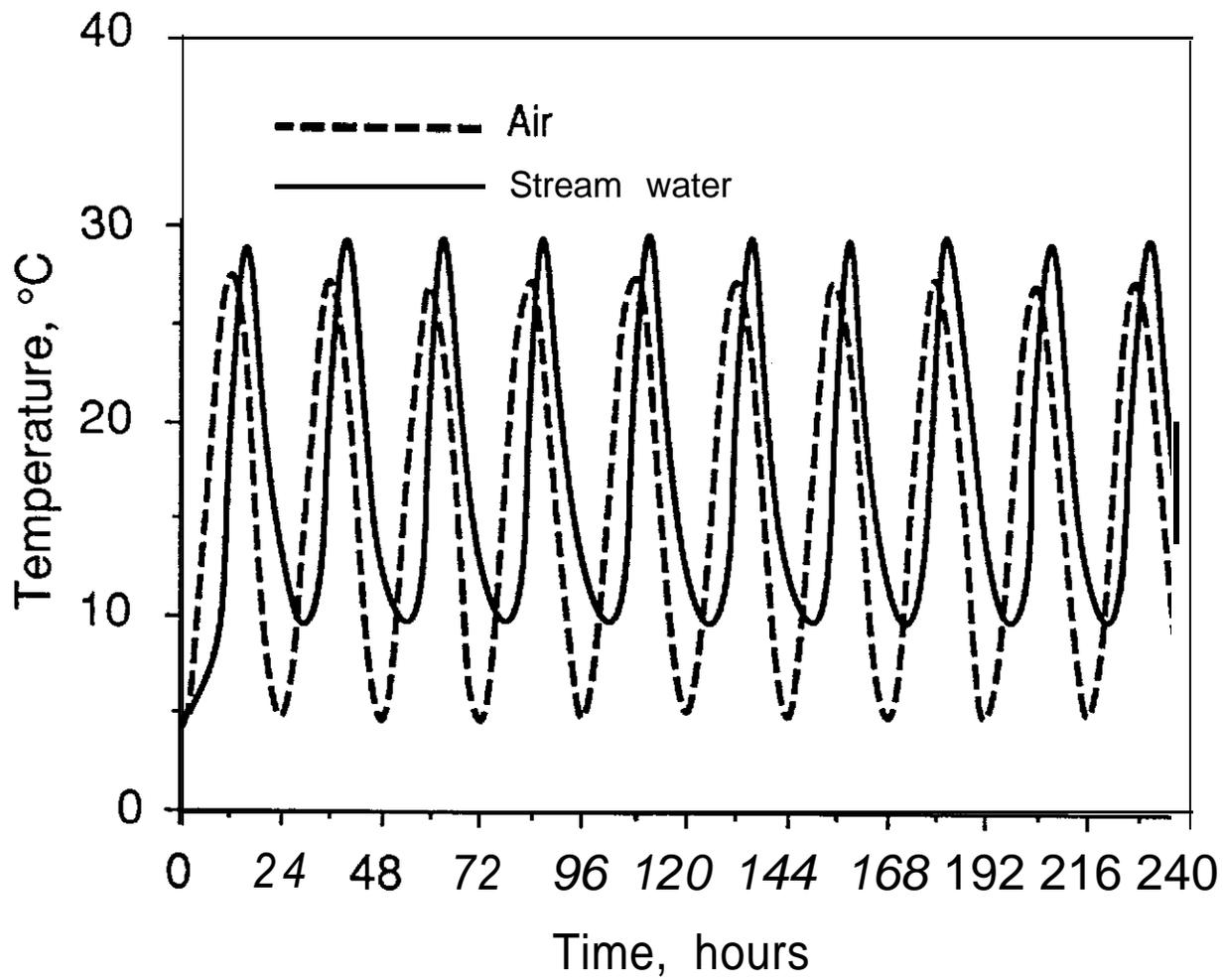
$h_c$  = convective heat transfer coefficient,  $W/(m^2 °C)$

$h_e$  = enthalpy of the evaporated water, J/kg

$h_{gw}$  = enthalpy of the groundwater, J/kg

hsb = convective heat **transfer** coefficient for the **streambed**,  $W/(m^2 °C)$

$h_w$  = enthalpy of the stream water, J/kg  
 $I_d$  = average daily solar insolation, W/m<sup>2</sup>  
 $k$  = streambed thermal conductivity, W/(m °C)  
 $k_e$  = convective mass transfer coefficient, kg/(m<sup>2</sup>•s•mbar)  
 $L_v$  = latent heat of vaporization of water, 2440x10<sup>3</sup> J/kg  
 $l$  = stream element length, m  
 $M$  = stream element mass, kg  
 $P(T_w)$  = saturation pressure of water vapor at  $T_w$ , mbar  
 $q_{conv}$  = heat flux due to convection, W/m<sup>2</sup>  
 $q_{net}$  = net heat flux, W/m<sup>2</sup>  
 $q_{sb}$  = streambed heat flux, W/m<sup>2</sup>  
 $q_{sky}$  = heat flux due to sky radiation, W/m<sup>2</sup>  
 $q_{solar}$  = heat flux due to solar radiation, W/m<sup>2</sup>  
 $q_{veg}$  = heat flux due to vegetation radiation, W/m<sup>2</sup>  
 $S$  = steady energy flux factor in, °C/s  
 $t$  = time, s  
 $T(x,t)$  = streambed temperature at depth  $x$  and time  $t$ , °K  
 $T_a$  = air temperature, °K  
 $T_{gw}$  = ground water temperature, °K  
 $TODF$  = time of day factor for solar radiation, dimensionless (see Equation 2)  
 $T_w$  = stream water temperature, °K  
 $U$  = energy transfer coefficient factor, s<sup>-1</sup>  
 $V_a$  = air velocity, m/s  
 $V_s$  = stream velocity, m/s  
 $x$  = depth below streambed surface, m  
 $\alpha$  = thermal diffusivity of the streambed, m<sup>2</sup>/s  
 $\alpha_{lw}$  = absorptivity of the stream for longwave radiation, unitless  
 $\alpha_{sw}$  = effective absorptivity of the stream for shortwave radiation, unitless  
 $\epsilon_w$  = water emissivity = absorptivity, a fraction  
 $\rho$  = water density, 1000 kg/m<sup>3</sup>  
 $\sigma$  = Stefan-Boltzmann's constant = 5.68x10<sup>-8</sup> W/(m<sup>2</sup> °K<sup>4</sup>)  
 $\nu$  = kinematic viscosity of water, m<sup>2</sup>/s  
 $\omega$  = angular frequency of variations, radians/s  
  
 $\bar{\quad}$  = indicates daily average value  
 $\prime$  = indicates fluctuating value about the average



*Figure 1- Air and predicted stream water temperature over a 10-day period for a stream of 0.1 m depth.*

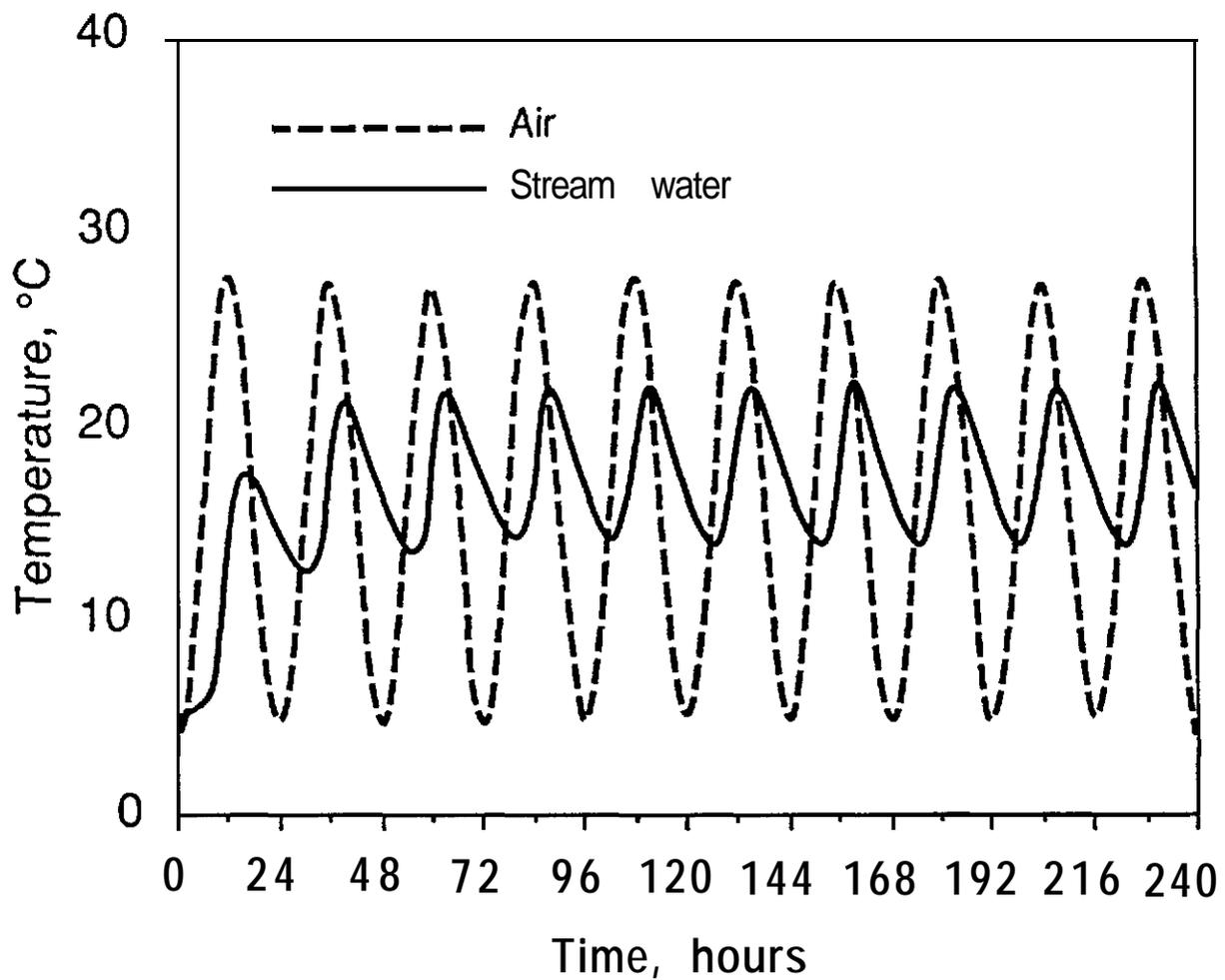
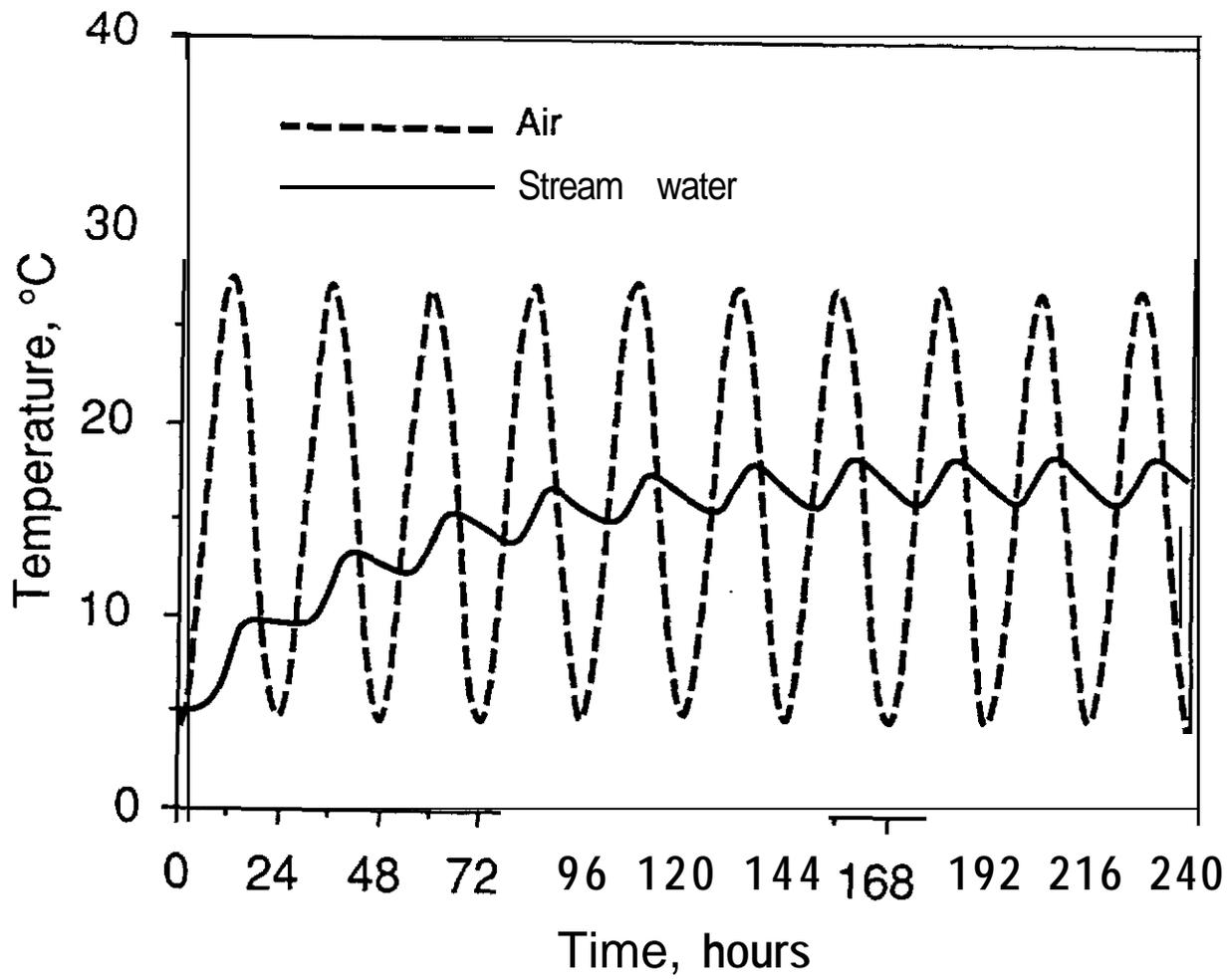


Figure 2 — Air and predicted stream water temperature over a 10-day period for a stream of 0.3 m depth.



*Figure 3— Air and predicted stream water temperature over a 10-day period for a stream of 1.0 m depth.*

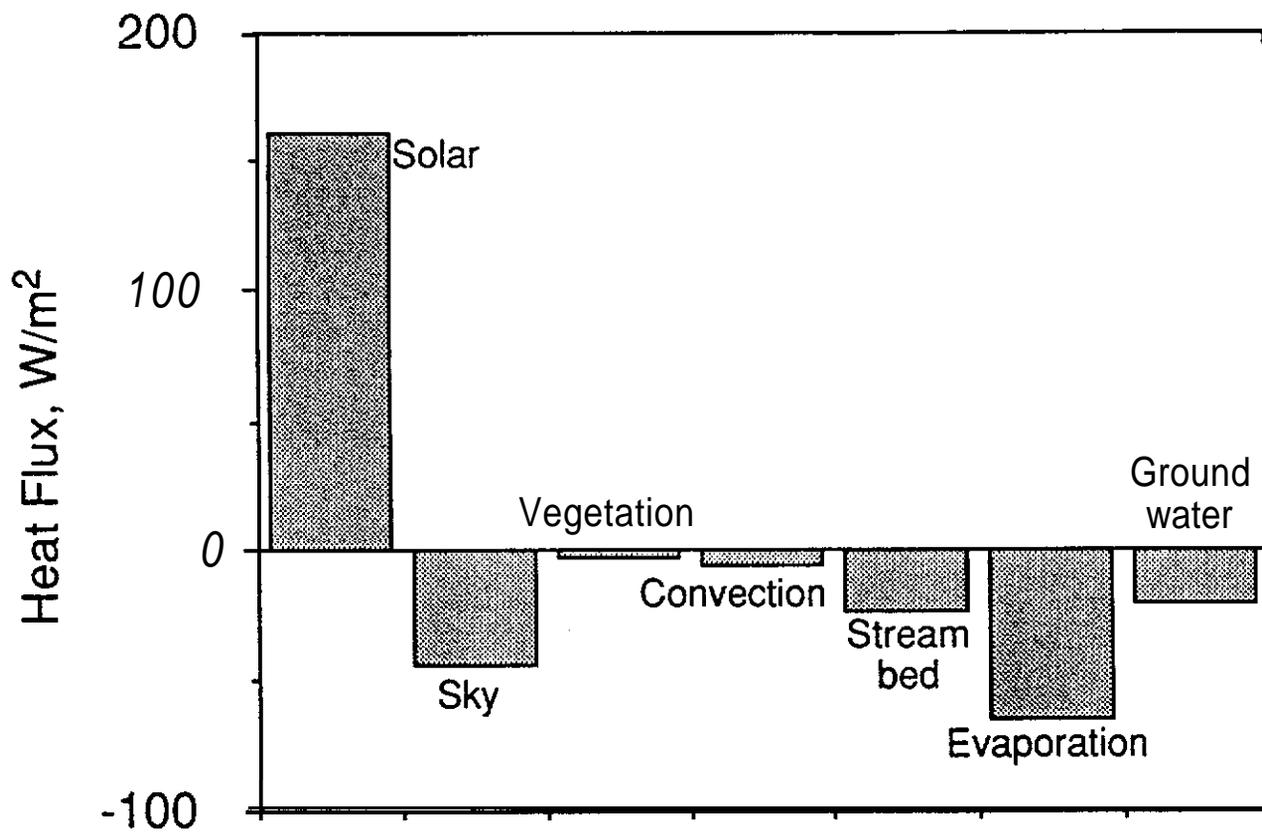


Figure 4 – Daily average heat flux to the stream due to the various energy transfer modes.

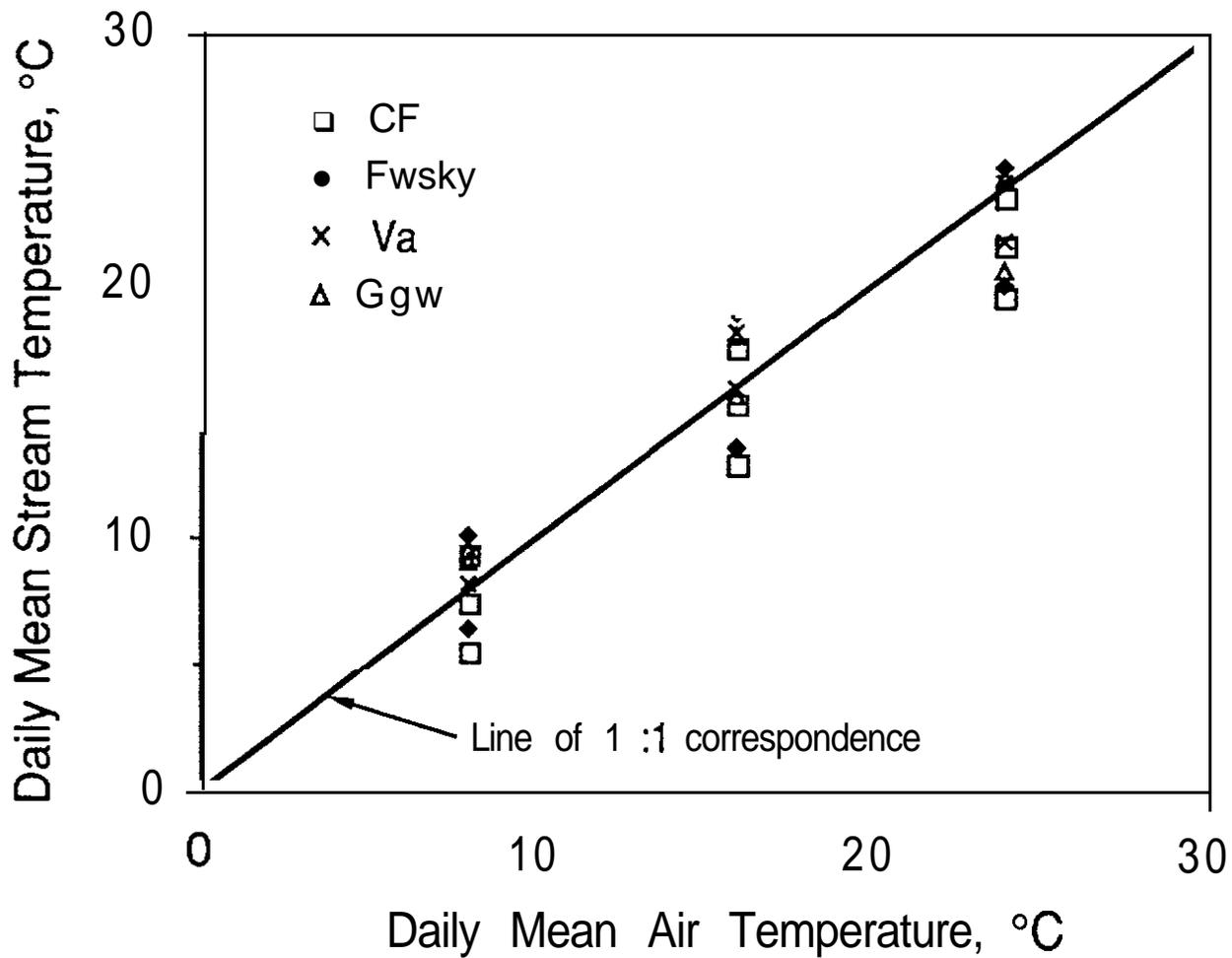
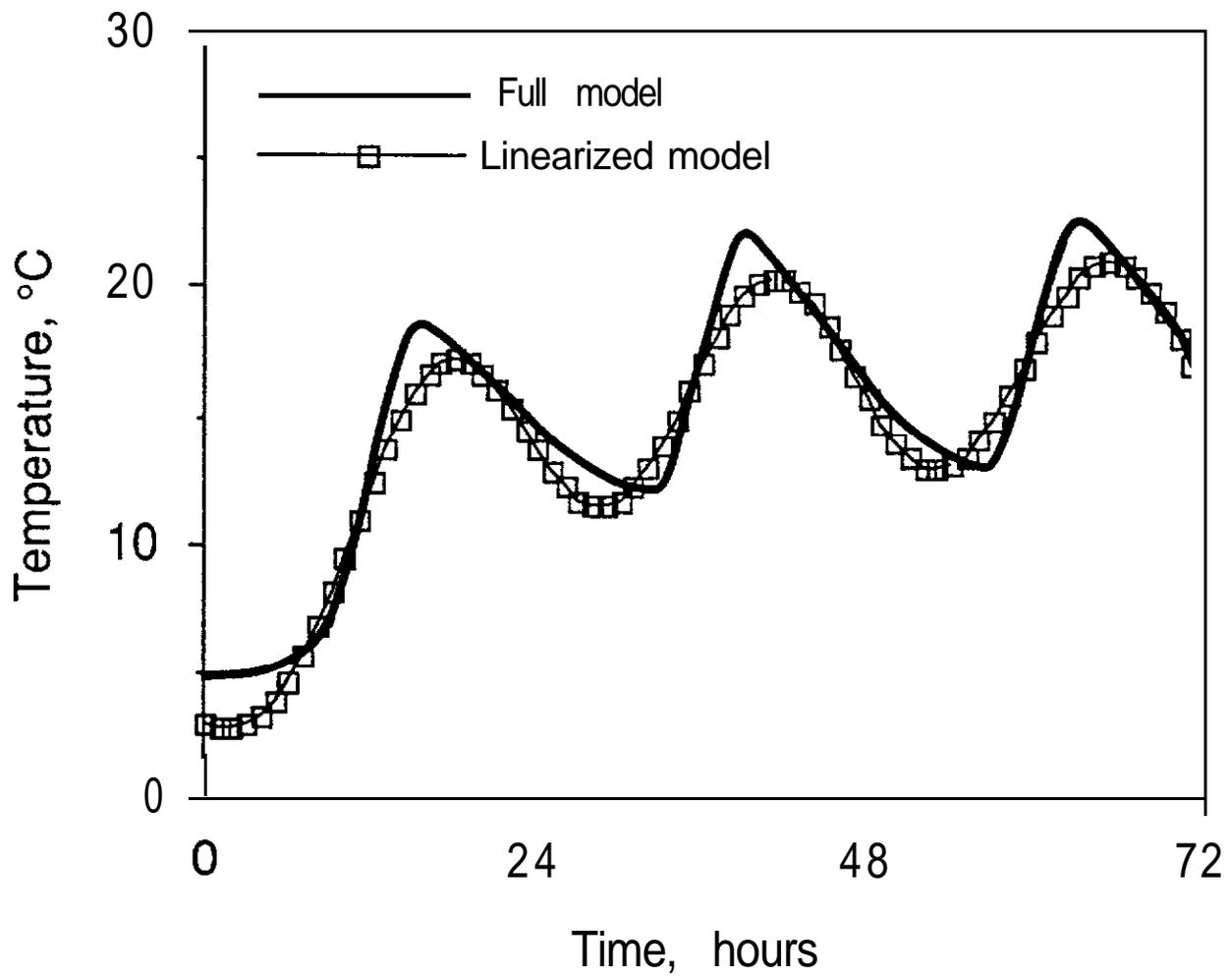
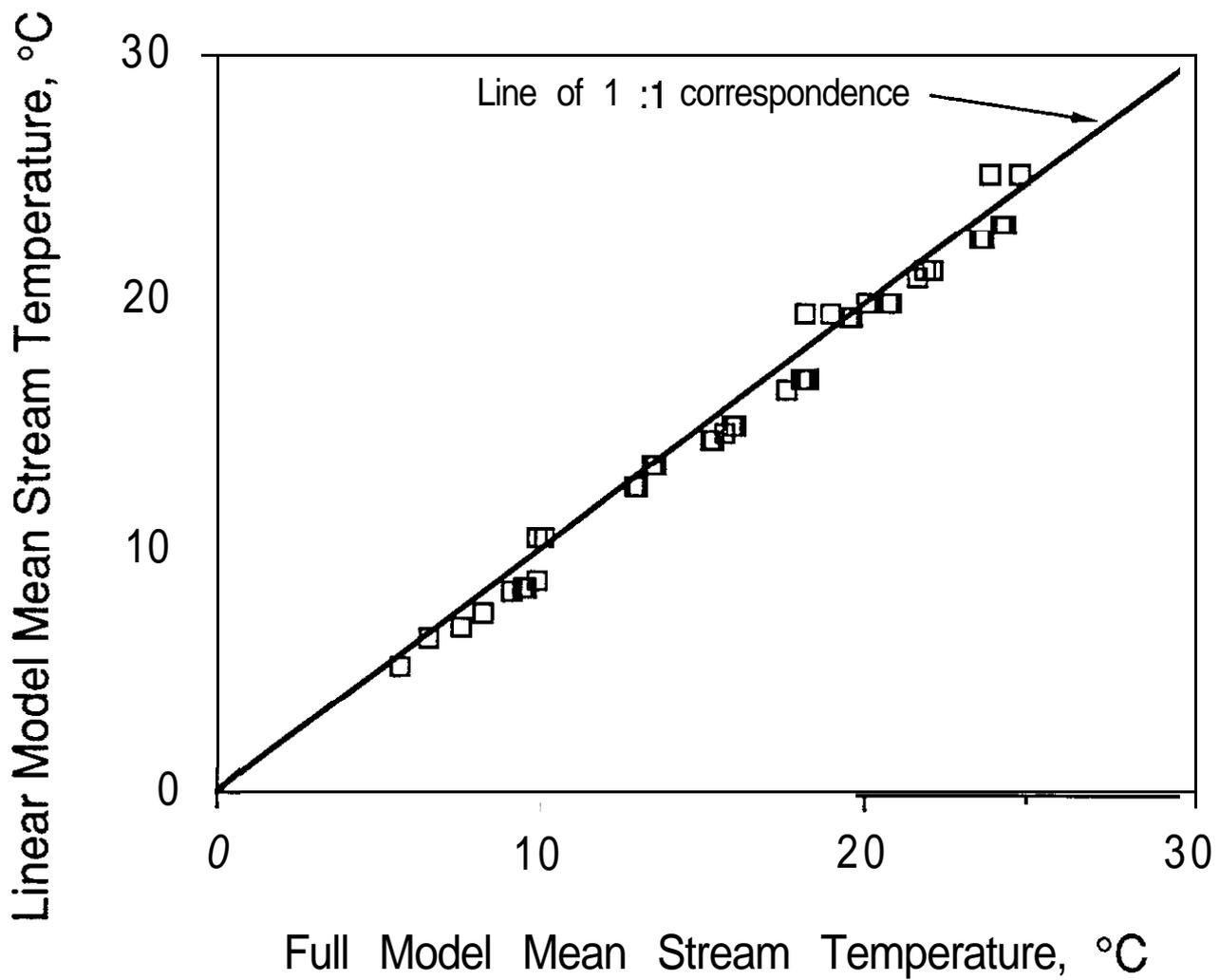


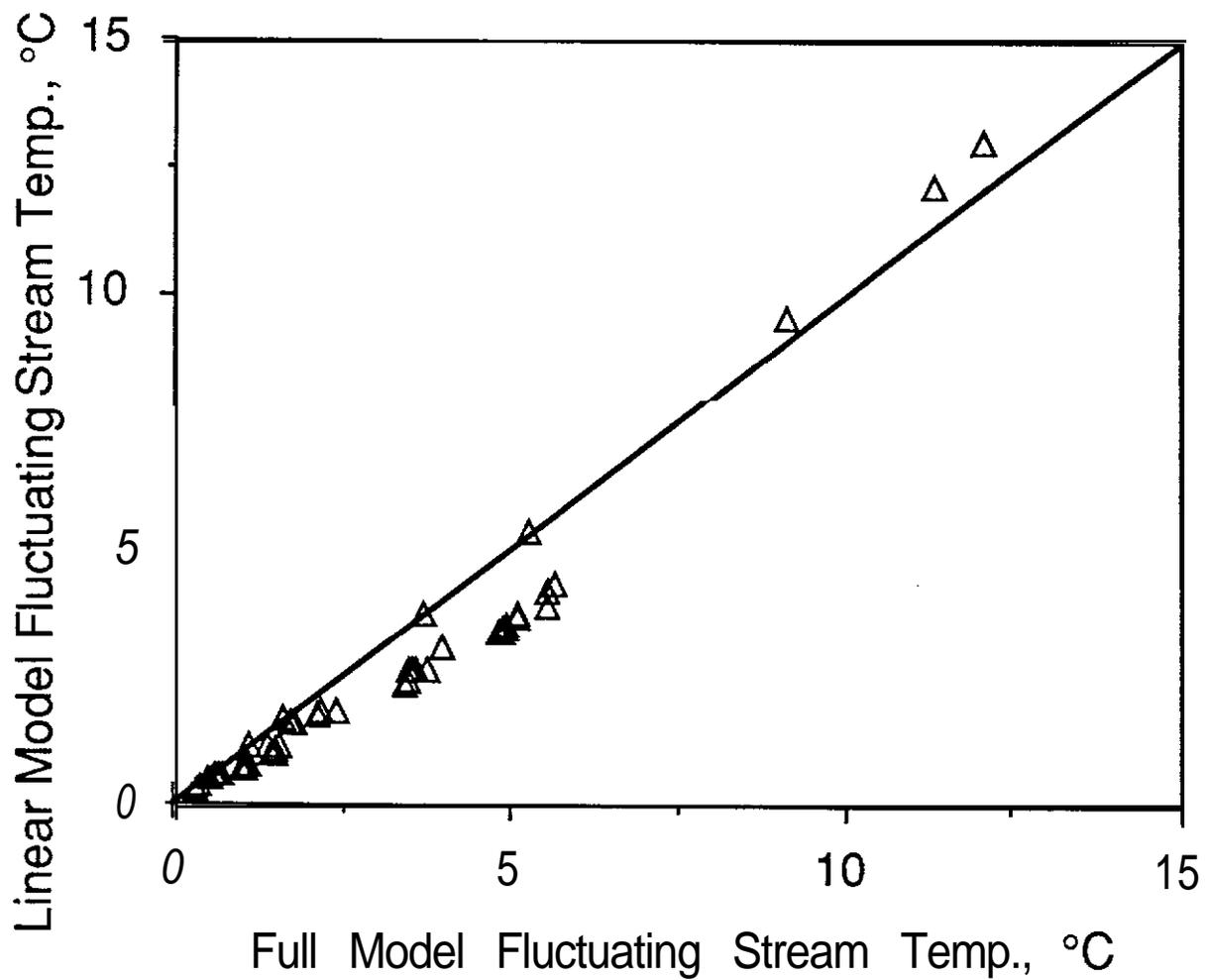
Figure 5 – Plot of predicted mean stream temperature as a function of the prescribed mean air temperature for a range of environmental conditions.



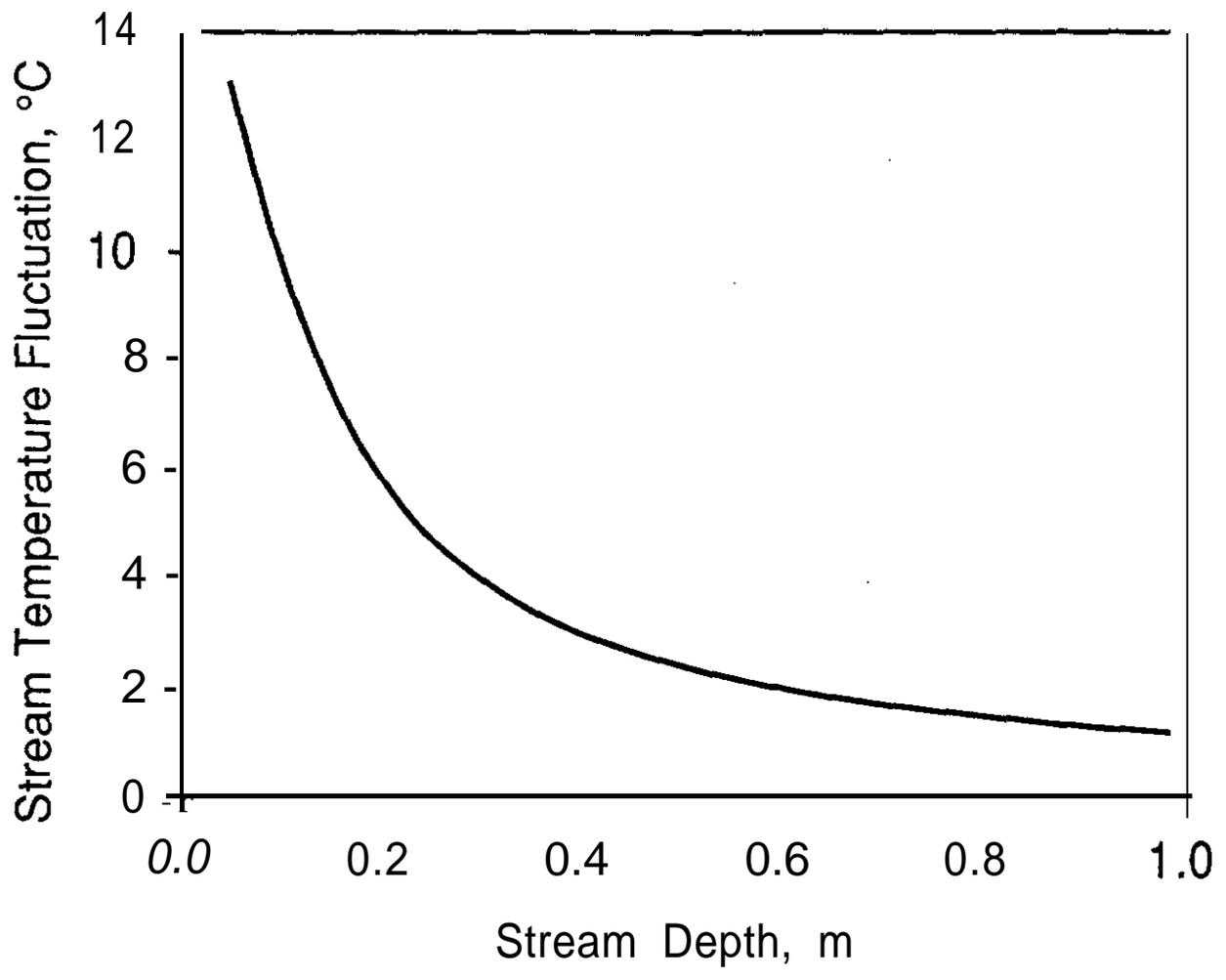
*Figure 6 – Comparison between the full stream-heating model and the linearized model.*



*Figure 7 – Plot of the predicted mean stream temperature for the linearized model versus that predicted by the full model for the same conditions.*



*Figure 8 – Plot of the predicted fluctuating stream temperature for the linearized model versus that predicted by the full model for the same conditions.*



*Figure 9 – Effect of stream depth on the magnitude of the stream temperature fluctuations.*